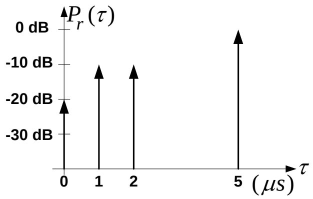
Example 13

- A multipath power delay profile is given below.



a) Calculate the mean excess delay.

b) Calculate the RMS delay spread.

c) Estimate the 50% coherence bandwidth of the channel.

d) Would this channel be suitable for AMPS or GSM service without the use of an equalizer?

a)

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k \phi(\tau_k)\tau_k}{\sum_k \phi(\tau_k)}$$

$$= \frac{(0.01)(0)+(0.1)(2)+(0.1)(1)+(1.0)(5)}{(0.01+0.1+0.1+1.0)} = 4.38\,\mu s$$

b)

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k \phi(\tau_k)\tau_k^2}{\sum_k \phi(\tau_k)}$$

$$= \frac{(0.01)(0)^2+(0.1)(2)^2+(0.1)(1)^2+(1.0)(5)^2}{(0.01+0.1+0.1+1.0)} = 21.07\,\mu s^2$$

The rms delay spread is given by

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2} = \sqrt{21.07 - (4.38)^2} = 1.37\,\mu s$$

c)

$$B_c \approx \frac{1}{5\sigma_\tau} = \frac{1}{5\times(1.37)} = 146\,kHz$$

d)

Since Bc is greater than 30 kHz, AMPS will work without an equalizer. However, GSM requires 200 kHz bandwidth which exceeds Bc. Thus an equalizer would be needed for the GSM.

Example 14

- Example on Delay PSD and Frequency-Correlation Function -
- Consider a WSSUS channel whose time-variant impulse response is given by

$$h(\tau,t) = \exp\left(-\frac{\tau}{T}\right) n(\tau)\cos(\Omega t + \Theta), \quad \tau \ge 0$$

where, T and Ω are constants, Θ is a random variable uniformly distributed in $[-\pi, \pi]$, and $n(\tau)$ is a real-valued random process independent of Θ, with $E[n(\tau)] = \mu_n$ and $E[n(\tau_1)\, n(\tau_2)] = \delta(\tau_1-\tau_2)$

a) Calculate the delay psd and the multipath delay spread.

b) Calculate the frequency correlation & channel coherence bandwidth.

c) Determine whether the channel exhibits frequency-selective fading for GSM systems with T = 0.1 ms.

a) Calculate the delay psd and the multipath delay spread

$$\phi_h(\tau) = F_{\Delta\tau}\left\{\frac{1}{2}E\left[h^*(\tau,t)h(\tau+\Delta\tau,t)\right]\right\}$$
$$= F_{\Delta\tau}\left\{\frac{1}{2}E\left[n(\tau)n(\tau+\Delta\tau)\right]E\left[\exp\left(-\frac{2\tau+\Delta\tau}{T}\right)\cos^2(\Omega t+\Theta)\right]\right\}$$
$$= F_{\Delta\tau}\left\{\frac{1}{4}\delta(\Delta\tau)\exp\left(-\frac{2\tau+\Delta\tau}{T}\right)E\left[1+\cos(2\Omega t+2\Theta)\right]\right\}$$
$$= \frac{1}{4}\exp\left(-\frac{2\tau}{T}\right)$$

where

$$E\left[\cos(2\Omega t+2\Theta)\right] = \int_{-\pi}^{\pi}\cos(2\Omega t+2\theta)\frac{1}{2\pi}d\theta = 0$$

For $\tau < 0$, $\phi_h(\tau) = 0$.

- The mean propagation delay is

$$\bar{\tau} = \frac{\int_0^\infty \tau\phi_h(\tau)d\tau}{\int_0^\infty \phi_h(\tau)d\tau} = \frac{\int_0^\infty \tau\frac{1}{4}\exp\left(-\frac{2\tau}{T}\right)d\tau}{\int_0^\infty \frac{1}{4}\exp\left(-\frac{2\tau}{T}\right)d\tau} = \frac{T}{2}$$

and the multipath delay spread is

$$T_m = \sigma_\tau = \left[\frac{\int_0^\infty (\tau-\bar{\tau})^2\phi_h(\tau)d\tau}{\int_0^\infty \phi_h(\tau)d\tau}\right]^{1/2}$$
$$= \left[\frac{\int_0^\infty (\tau)^2\phi_h(\tau)d\tau}{\int_0^\infty \phi_h(\tau)d\tau} - (\bar{\tau})^2\right]^{1/2} = \frac{T}{2}$$

b) The frequency correlation function is

$$\Phi_H(\Delta f) = F\left[\phi_h(\tau)\right]$$
$$= \int \frac{1}{4}\exp\left(-\frac{2\tau}{T}\right)e^{-j2\pi(\Delta f)\tau}\,d\tau$$
$$= \frac{T}{8+j8\pi T(\Delta f)}$$

The coherence bandwidth is

$$(\Delta f)_c \approx \frac{1}{T_m} = \frac{2}{T}\ s$$

c) With T = 0.1 ms, we have $(\Delta f)_c$ = 20 kHz. The GSM channels have a bandwidth of 200 kHz. Since $(\Delta f)_c$, << 200 kHz, the channel fading is frequency selective.

Example 15 (Doppler PSD)

- For the channel specified in Example 16 with $\Omega = 10\pi$
  - a) the Doppler psd,
  - b) the mean Doppler shift and the rms Doppler spread,
  - c) the channel coherence time, and
  - d) whether the channel exhibits slow fading for GSM systems

a) The Doppler psd can be calculated by taking the Fourier transform of the time correlation function $\phi_H(\Delta t)$. In this case, we need to calculate the correlation function $\phi_h(\tau,\Delta t)$ first.

For the WSS channel, we have

$$\phi_h(\tau,\Delta t) = F_{\Delta\tau}\left\{\frac{1}{2}E\left[h^*(\tau,t)h(\tau+\Delta\tau,t+\Delta t)\right]\right\}$$

$$= F_{\Delta\tau}\left\{\frac{1}{2}E\left[\begin{array}{l}\exp\left(-\frac{\tau}{T}\right)n(\tau)\cos(\Omega t+\Theta)\\ \times\exp\left(-\frac{\tau+\Delta\tau}{T}\right)n(\tau+\Delta\tau)\cos(\Omega t+\Omega\Delta t+\Theta)\end{array}\right]\right\}$$

$$= F_{\Delta\tau}\left\{\begin{array}{l}\frac{1}{4}\exp\left(-\frac{2\tau+\Delta\tau}{T}\right)E\left[n(\tau)n(\tau+\Delta\tau)\right]\\ \times E\left[\cos(\Omega\Delta t)+\cos(2\Omega t+\Omega\Delta t+2\Theta)\right]\end{array}\right\}$$

$$= F_{\Delta\tau}\left\{\frac{1}{4}\delta(\Delta\tau)\exp\left(-\frac{2\tau+\Delta\tau}{T}\right)\cos(\Omega\Delta t)\right\}$$

$$= \frac{1}{4}\exp\left(-\frac{2\tau}{T}\right)\cos(\Omega\Delta t)$$

The time correlation function is then

$$\phi_H(\Delta t) = \phi_H(\Delta f,\Delta t)\big|_{\Delta f} = 0$$

$$= \int_{-\infty}^{\infty}\phi_h(\tau,\Delta t)\,d\tau$$

$$= \frac{1}{4}\cos(\Omega\Delta t)\int_0^{\infty}\exp\left(-\frac{2\tau}{T}\right)d\tau$$

$$= \frac{T}{8}\cos(\Omega\Delta t)$$

The Doppler psd is

$$\Phi_H(v) = F\left[\phi_H(\Delta t)\right]$$

$$= F\left[\frac{T}{8}\cos(\Omega\Delta t)\right]$$

$$= \frac{T}{16}\left[\delta(2\pi v-\Omega)+\delta(2\pi v+\Omega)\right]$$

Channel introduces 2 Dopplar shifts at $\pm\Omega/2\pi = \pm 5$ Hz

b) The mean Doppler shifts is zero as the two are opposite of each other and have the same psd. This can also be verified using the equation below and the psd computed above

$$\bar{v} = \frac{\int_{-\infty}^{\infty} v\Phi_H(v)\,dv}{\int_{-\infty}^{\infty}\Phi_H(v)\,dv}$$

The rms Doppler spread

$$\sigma_v = \sqrt{\frac{\int_{-\infty}^{\infty}(v-\bar{v})^2\,\Phi_H(v)\,dv}{\int_{-\infty}^{\infty}\Phi_H(v)\,dv}}$$

$$= \left[\frac{\int_{-\infty}^{\infty}(v)^2\,\frac{T}{16}\left[\delta(2\pi v-\Omega)+\delta(2\pi v+\Omega)\right]dv}{\int_{-\infty}^{\infty}\frac{T}{16}\left[\delta(2\pi v-\Omega)+\delta(2\pi v+\Omega)\right]dv}\right]^{1/2}$$

b) The rms Doppler spread

$$\sigma_v = \left[\frac{\frac{T}{16}\left[\left(\frac{\Omega}{2\pi}\right)^2+\left(-\frac{\Omega}{2\pi}\right)^2\right]}{\int_{-\infty}^{\infty}\frac{T}{16}[1+1]}\right]^{1/2} = \frac{\Omega}{2\pi} = 5\,Hz$$

c) Coherence Time

$$(\Delta t)_c \approx \frac{1}{\sigma_v} = 0.2\ s$$

d) In GSM systems, the data rate is R_s = 270.833 kbps, which corresponds to a symbol interval of

$$T_s = \frac{1}{R_s} \approx 3.7\times10^{-6}\ s$$

Since $T_s \ll (\Delta t)_c$, the channel exhibits slow fading

b) The mean Doppler shifts is zero as the two are opposite of each other and have the same psd. This can also be verified using the equation below and the psd computed above

$$\overline{v} = \frac{\int_{-\infty}^{\infty} v\Phi_H(v)\,dv}{\int_{-\infty}^{\infty} \Phi_H(v)\,dv}$$

b) Calculate the frequency correlation & channel coherence bandwidth.

c) Determine whether the channel exhibits frequency-selective fading for GSM systems with T = 0.1 ms.

Example xxx

- In a transmission and reception of signals to and from moving vehicles, the transmitted signal frequency is shifted in direction proportional to the speed of the vehicle. Suppose that a vehicle is traveling at a speed of 100 Km/hr relative to the base station in a cellular communications system. The signal is narrowband and transmitted at a carrier frequency of 1 GHz

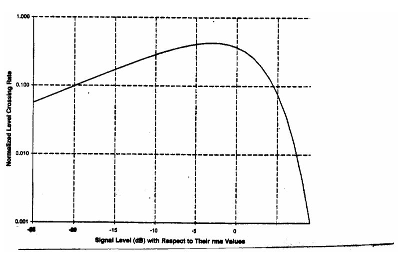
a) Determine the Doppler frequency shift

b) What should be the bandwidth of a Doppler frequency tracking loop if the loop is designed to track Doppler frequency shift of vehicles traveling at 100 Km/hr?

c) Suppose the transmitted signal bandwidth is 2 MHz centered at 1 GHz, determine the Doppler frequency spread between the upper and lower frequencies in the signal

Example 16

- A vehicle is traveling at 24 km/hr in a cellular system operating at 900 MHz. A threshold signal level is set at -20 dB and the measured signal level is shown below.



a) What is the Level Crossing Rate (LCR) at the threshold level if the fading amplitude is

(i) Rayleigh model, (ii) Ricean model, *K = 10* dB

b) What is the Average Duration of Fades (ADF) at the threshold level if the fading envelope is

(i) Rayleigh model, (ii) Ricean model, *K = 3* dB

$$v = 24\,km/hr = \frac{24\times10^3 m}{3600\,sec} = 6.67\ m/s$$

$$f_c = 900\,MHz \Rightarrow \lambda = \frac{3\times10^8 m}{900\times10^6} = 0.33\ m$$

$$\Rightarrow f_m = \frac{v}{\lambda} = \frac{6.67}{0.33}Hz = 20\,Hz$$

The normalized LCR, n_R is $n_R = \rho e^{-\rho^2}$ **and is given in the Fig.**

a) (i) Rayleigh Model

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2} = \sqrt{2\pi} f_m n_R$$

From the Fig. at –20 dB signal level, $n_R = 0.1$

$$N_R = \sqrt{2\pi}\times20\times0.1 = 5.01 \text{ crossing/sec}$$

(ii) Rician Model

$$K = 10\,dB = 10$$

$$\rho = \frac{R}{R_{rms}} = \frac{0.1}{\sigma\sqrt{2}} = \frac{0.707}{\sigma}$$

Assuming σ = 1, then ρ = 0.707

$$N_R = \sqrt{2\pi(K+1)}\,f_m \rho e^{-K-(K+1)\rho^2} I_0\left(2\rho\sqrt{K(K+1)}\right)$$

$$= \sqrt{22\pi}\times 20\times(0.707)\,e^{-10-11(0.707)^2} I_0\left(2\times 0.707\sqrt{110}\right)$$

$$= 2.185\times 10^{-5} I_0(14.83)$$

$$= 2.185\times 10^{-5}\times\left(2.8822\times 10^{5}\right)$$

$$= 6.297 \text{ crossing/sec}$$

b) (i) Rayleigh Model

$$\overline{\tau} = \frac{e^{\rho^2}}{\rho f_m\sqrt{2\pi}} = \frac{e^{(0.707)^2}}{0.707\times 20\times\sqrt{2\pi}} = \frac{0.6485}{34.444} = 18.3\,ms$$

(ii) Rician Model

$$\overline{\tau} = \frac{1-Q\left(\sqrt{2\pi K},\sqrt{2(K+1)\rho^2}\right)}{\sqrt{2\pi(K+1)}\,f_m\rho e^{-K-(K+1)\rho^2} I_0\left(2\rho\sqrt{K(K+1)}\right)}$$

$$\downarrow K = 3dB = 2$$

$$= \frac{1-Q\left(\sqrt{4\pi},\sqrt{6\times(0.707)}\right)}{\sqrt{6\pi}\times 20\times 0.707\times e^{-2-3(0.707)^2} I_0\left(1.414\sqrt{6}\right)}$$

$$= \frac{1-Q(3.545,1.732)}{1.855\times I_0(3.464)} = \frac{1-Q(3.545,1.732)}{1.855\times 7.1584}$$

$$= 0.075\left[1-Q(3.545,1.732)\right]$$